

The citation for this article is Mallama, A., Sôma, M., Sada, P.V., Modic, R.J. and Ellington, C.K. (2009) *Icarus* 200 265–270. DOI 10.1016/j.icarus.2008.11.022

## **Astrometry of Iapetus, Ariel, Umbriel, and Titania from Eclipses and Occultations**

### Abstract

Highly accurate astrometric positions obtained from eclipses and occultations of planetary satellites are reported. These measurements may be used to test existing ephemerides, to improve upon them, and to fit system constants such as satellite masses and planetary zonal harmonics.

Eclipse and occultation photometry of 5 uranian satellite mutual events has resulted in precise astrometry for 3 of these moons. Relative satellite positions were determined with an uncertainty of less than 10 milli-arcseconds for 4 of the events. These observations plus two additional data from C. Miller and N. J. Chanover (private communication) indicate that predictions based on the SPICE (Acton, 1996) ephemeris URA083 and those from the LA06 ephemeris in a paper by Arlot et al. (2006) are significantly more accurate than predictions generated by Christou (2005) using the GUST86 ephemeris in the along-track component of motion. The observations indicate that Ariel, Umbriel and Titania are lagging behind their predicted positions for all of the ephemerides, but by varying distances and significance levels. Analysis of data recorded by Hidas et al. (2008) suggests a similar lag for Oberon.

Photometry recorded during the ingress portion of a saturnian eclipse of Iapetus on 2007 May 5 indicates that the middle of the event occurred at geocentric UTC 02:14:58. At that moment the center of the satellite disk facing the Sun was intersected by a solar-centered ray refracted at a minimum altitude of 240 km above the 1-bar pressure level in the planet's atmosphere. The uncertainty in the timings due to observational scatter was only 5 s which equates to 16 km of

lapetus motion, but other factors increased the overall uncertainty to 111 km or 16 milli-arcseconds at the distance of Saturn from the Sun. The astrometric result is fit very well by the SPICE ephemeris SAT288.

Key words: 'Uranus, satellites', 'lapetus', 'eclipses', 'occultations', 'orbit determination'

## 1. Introduction

Precise astrometric satellite positions can be derived from ground-based photometry of mutual eclipses and occultations involving two planetary satellites, as well as from eclipses of individual satellites by their planets. Methods of observing these phenomena and applying synthetic light curves to solve for astrometric parameters have been described by Mallama (1992a and 1992b). These studies and others (e.g., Vasundhara et al., 2003; Thuillot and Descamps, 1999; and Thuillot et al., 1996) demonstrate that astrometry of milli-arcsecond (mas) accuracy can be achieved.

Astrometric measurements may be used to test the ephemerides of planetary satellites. They also serve as observational input for the generation of new, more accurate ephemerides and for improvement of the fits to satellite masses, planetary zonal harmonics, and other system constants. Thus, photometric astrometry data are important to planetary science because they provide critical information of the highest accuracy obtainable from the Earth.

In Section 2 of this paper astrometric results are derived from observations of several mutual events involving the uranian satellites Ariel, Umbriel and Titania, and they are compared with three sets of uranian satellite ephemerides. Section 3 presents observations obtained during a recent eclipse of Iapetus by Saturn. The time of mid-ingress is derived and it is compared to the predictions of four ephemerides. Section 4 summarizes the results.

There are very few opportunities to observe the phenomena of these satellites because they only occur at intervals of one-half of their planet's orbital period. Thus, the next series of uranian satellite mutual events will be centered in 2050 and the next eclipse of Iapetus will be in 2022.

## 2. Uranian Satellites

Hammel (2006) listed satellite eclipses and occultations among the many phenomena to be observed during the passage of Uranus through its equinox in 2007. Two sets of predictions for the mid-times and minimum satellite separations (impact parameters) of these events were published. Christou (2005) used the JPL URA027 ephemeris for the satellite vectors which is based on the GUST86 semi-analytical ephemeris of Laskar and Jacobson (1987). Arlot et al. (2006) used the LA06 ephemeris (Lainey, 2008) which comes from the Numerical Orbit and Ephemeris software developed at the Institut de Mécanique Céleste et de Calcul des Ephémérides (IMCCE). Arlot et al. (2006) assess the accuracy of GUST86 as 100 mas and that of LA06 as 50 mas. At the mean distance to Uranus these correspond to 1,400 km and 700 km, respectively.

A comprehensive set of mutual event observations is being collected by the IMCCE, <http://www.imcce.fr/pheura07>, and will be analyzed in a few years from now. However, the smaller data set presented here shows that some conclusions can already be drawn and it predicts other results that can be tested when the full data set has been studied.

### 2.1 Modeling and data analysis

Synthetic mutual event light curves depend on the interplanetary geometry of the event, the bi-directional reflectance (BDR) function of the satellite being eclipsed or occulted, and solar limb-darkening. The geometrical parameterization is solved for when the observations are fitted to the data, and the solar limb-darkening effect is exceedingly small at the distance of Uranus. The uranian satellites do not exhibit strong hemispheric albedo differences which would, otherwise, be evident from their rotational light curves (see Figure 4 of Karkoshka 2001). Likewise, their limb-darkening is hardly noticeable since the maximum phase angle of Uranus is only  $3^\circ$ . Mallama (2007) determined that BDR effects can displace uranian satellite photocenters by only about 10

km. These effects are negligible in comparison to the magnitudes of the expected ephemeris errors noted above.

The relative brightness between the two satellites has to be taken into account in order to fit the synthetic light curve of an occultation. Additionally, when the angular distance between eclipsing satellites does not allow them to be measured separately, the synthetic light curve must be adjusted for the relative satellite brightness as well. The R-band satellite magnitudes from Karkoschka (2001) as listed by Christou (2005) were used to compute these brightness ratios. This band pass approximates the response of the red-sensitive unfiltered CCDs used for recording the uranian events.

## 2.2 Observational results

Photometric data were obtained for 3 occultations and 2 eclipses involving the satellites Ariel, Umbriel and Titania. Two examples of synthetic light curves fitted to observations demonstrate the two types of events. Fig. 1 shows the flat bottom of an annular event and Fig. 2 illustrates the sharply pointed minimum light of a partial event. The shoulders of these light curves also demonstrate that an occultation begins and ends very abruptly while the changes are more gradual for an eclipse because of the penumbral shadow.

Table 1 lists the two critical parameters of each light curve solution, viz., the geocentric mid-event time, and the angular separation between the two satellites as seen from the Earth for occultations and from the Sun for eclipses. These parameters were transformed to Right Ascension and Declination differences based on the direction of relative motion between the satellites.

The depth of the synthetic light curve for the annular occultation of August 15 is not significantly different for either a central annular occultation or one at the limit between an annular and a partial event. Therefore the separation could only be loosely determined as  $0 \pm 20$  mas. The parameters

for the other mutual events were determined much more accurately having estimated uncertainties from 1 to 7 mas.

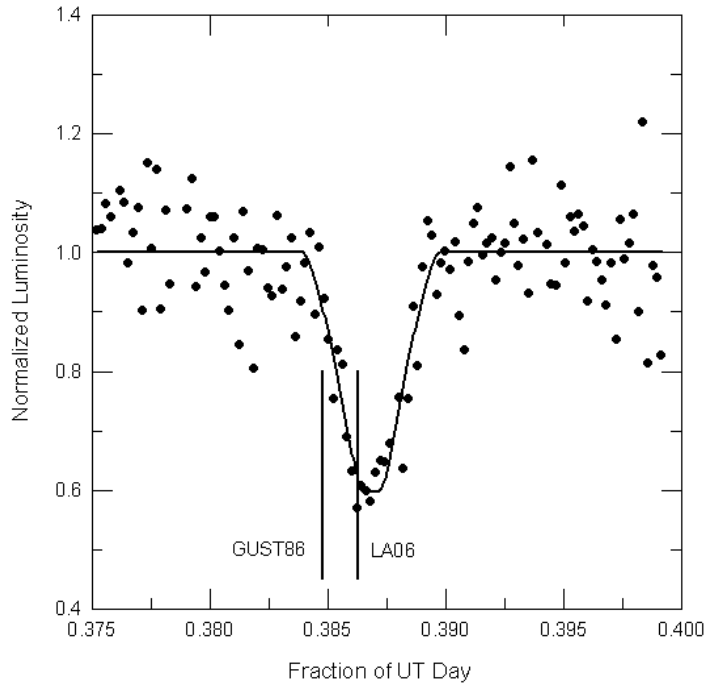


Fig. 1. Annular occultation of Titania by Umbriel observed on 2007-08-15. The vertical lines show the predicted times of mid-event for the two ephemerides indicated.

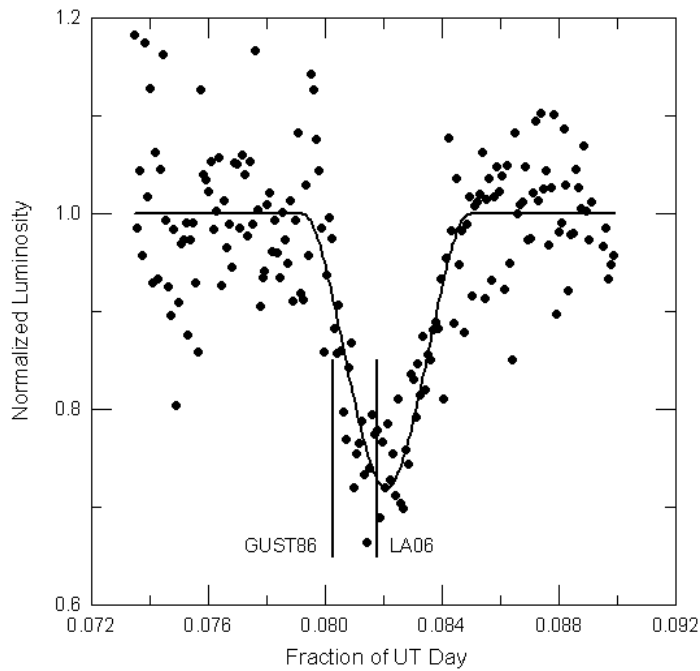


Fig. 2. Partial eclipse of Titania by Umbriel observed on 2007-12-08.

Table 1. Summary of uranian satellite observations and astrometry

#	Date of 2007	Event type*	Mid-event UTC	Separation mas	delta RA** mas	delta Dec** mas	Observer ***
1	August 13	1 O 2 P	03:05:41 (5)	34 ( 6)	-33 ( 6)	-9 ( 2)	Modic
2	August 15	2 O 3 A	09:17:04 (32)	0 (20)	0 (19)	0 (15)	Ellington
3	August 19	2 O 1 P	07:59:23 ( 9)	14 ( 2)	-13 ( 2)	-3 ( 2)	Sada
4	December 7	1 E 2 P	03:33:22 (12)	19 ( 5)	-18 ( 5)	-5 ( 7)	Sada
5	December 8	2 E 3 P	01:58:13 ( 4)	36 ( 3)	+34 ( 3)	+11 ( 1)	Sada

Notes for Table 1:

- \* satellite: 1-Ariel, 2-Umbriel, 3-Titania
- \* type of obscuration: O-occultation, E-eclipse
- \* degree of obscuration: P-partial, A-annular
- \*\* coordinate of the eclipsing or occulting satellite minus the other
- \*\* delta RA has been corrected for cosine of declination
- \*\*\* observer information (latitude, longitude, telescope aperture):  
 Ellington, 47.360 N, 122.160 W, 25 cm  
 Modic, 41.547 N, 81,.081 W, 40 cm  
 Sada, 25.623 N, 100.346 W, 36 cm  
 (numbers in parentheses are uncertainties)

### 2.3 Comparison with ephemerides

The O – Cs of the astrometric results for the five uranian mutual events are useful for assessing ephemeris accuracy. Besides testing the published predictions of GUST86 and LA06, the SPICE (Acton, 1996) system was run with orbit kernel URA083 (prepared by R. A. Jacobson) in order to evaluate that ephemeris as well. Columns 2, 3 and 4 of Table 2 clearly distinguish between the ephemerides since the root-mean-square (RMS) O – C residuals of mid-event times versus the newer ephemerides, 34 s for LA06 and 41 s for URA083, are much smaller than the 134 s RMS for GUST86. In the next three columns the O – Cs have been converted to mas based on the product of the components of satellite motion in the plane of the sky times the residuals in seconds. The final three columns show that the O – Cs of the impact parameters for the newer ephemerides are also less than for GUST86 although the improvement is more modest.

Table 2. O – C Residuals for GUST86, LA06 and URA083

#	Mid-event (s)			Mid-event (mas)			Impact parameter (mas)		
	GUST86	LA06	URA083	GUST86	LA06	URA083	GUST86	LA06	URA083
1	+ 91	+ 8	-4	24	2	1	+14	+20	+10
2	+177	+51	+69	66	20	26	---	---	---
3	+115	+11	-11	22	2	2	-51	-22	-10
4	+116	+48	+35	15	6	4	-28	+ 5	+14
5	+152	+24	+49	59	10	20	+ 6	-18	+12
<b>RMS</b>	<b>134</b>	<b>34</b>	<b>41</b>	<b>43</b>	<b>11</b>	<b>15</b>	<b>29</b>	<b>18</b>	<b>12</b>

The overall positional O – C is the root-sum-square (RSS) of the mid-event O – C in mas and the impact parameter O – C in mas for each ephemeris. These RSS results are 52 mas for GUST86 and 21 mas for LA06, and they indicate that both ephemerides are about twice as accurate as Arlot et al. (2006) had expected. However, it should be noted that those authors referred to all five large Uranian moons including Miranda for which the ephemeris may be much less accurate. The overall positional O – C for URA083, 19 mas, is similar to that of LA06.

#### 2.4 Orbital lag

Since mutual event observations indicate the relative positions of two satellites, their individual along-track locations can only be derived by solving a set of linear equations. The general formula for mutual eclipse O – C ephemeris residuals is

$$(O - C)_k = (L_J * \text{Cos}(\theta_{JK}) - L_I * \text{Cos}(\theta_{IK})) / (V_J * \text{Cos}(\theta_{JK}) - V_I * \text{Cos}(\theta_{IK}))$$

Eq. 1



where  $O - C$  is the observed quantity,  $L$  is the along-track positional lag of a satellite behind its ephemeris,  $\theta$  is the orbital longitude of the satellite measured relative to superior conjunction with the Sun (for an eclipse) or the Earth (occultation),  $V$  is the satellite orbital velocity, the indices  $I$  and  $J$  distinguish the satellites Ariel, Umbriel and Titania, and  $K$  refers to the observed events. The right-hand side of the equation is essentially distance divided by velocity which yields time. The  $L \cdot \cos(\theta)$  terms are the satellite positional errors projected onto the plane of the sky, and the  $V \cdot \cos(\theta)$  terms are the satellite velocities projected on the sky.

C. Miller and N. J. Chanover (private communication) sent us two additional mid-event times for mutual events of Ariel, Umbriel and Titania, which add to the robustness of our solution. The along-track values listed in Table 3 are the result of solving all seven equations.

Table 3. Orbital lag, uncertainty and significance.

	GUST86	LA06	URA083
Ariel:			
Lag (km)	+296	+ 350	+ 228
Uncertainty (km)	+/- 96	+/- 126	+/- 177
Significance (sigma)	3.1	2.8	1.3
Umbriel:			
Lag (km)	+516	+103	+59
Uncertainty (km)	+/- 34	+/- 30	+/- 31
Significance (sigma)	15.2	3.4	1.9
Titania:			
Lag (km)	+681	+155	+308
Uncertainty (km)	+/- 76	+/- 54	+/- 46
Significance (sigma)	9.0	2.9	6.7
Initial RMS of $O - C$ (s)	135.1	31.2	39.1
RMS after adjustment (s)	7.9	13.4	14.0

The positive values of ephemeris residuals indicate that the satellites all appear to be lagging behind their predicted locations. For GUST86 the lag varies from 296 km to 681 km and they are significant for all three satellites. The lag distances are considerably smaller for LA06, ranging from 103 to 350 km and only Umbriel is significant at the level of 3 sigma. For URA083 the lag distances run from 59 to 308 km and only that of Titania is significant. Since this is primarily an observational paper the lag distances are not analyzed any further. However we note that the statistically significant results may point to missing perturbations (for example, uranian precession or tidal effects) in some of the satellite ephemerides.

As shown in the last two rows of Table 3 along-track adjustment reduces the RMS of the O – C dramatically for all three ephemerides. Taking only the five mutual events reported in this paper, the RMS values after adjustment (10.8 s for GUST86, 12.8 s for LA06 and 9.2 s for URA083) are comparable to the RMS of the observational timing uncertainties listed in Table 2. The RMS of all 5 of those uncertainties is 16.1 s, while removing the large observational uncertainty of event 2 from consideration reduces the RMS to 8.1 s.

## 2.5 Discussion of other uranian satellite astrometry

The reviewers of this paper pointed out an occultation of Umbriel by Oberon observed by Hidas et al. (2008). The time of mid-event was late by 141 s relative to GUST86 and by 36 s relative to both LA06 and URA083. These O – Cs and the lag distances derived above for Umbriel were input to Eq. 1 in order to compute the following along-track lags of Oberon: +582 +/- 42 km for GUST86, +174 +/- 40 km for LA06, and +213 +/- 41 km for URA083. Thus Oberon also appears to be lagging behind all three ephemerides.

Conventional astrometry of the uranian satellites was obtained by Veiga and Vieira Martins (1999) between 1982 and 1998 and by Shen et al. (2002) from 1995 through 1997. They derived

residuals with respect to GUST86 with standard deviations on the order of 30 to 50 mas for Ariel, Umbriel and Titania. At the mean opposition distance to Uranus those residuals corresponds to 400 - 660 km, which is approximately as large as the lag distances for that ephemeris derived in this paper. Veiga and Vieira Martins chose to compute their residuals as delta X and delta Y values while Shen et al. chose separation and position angle. If their data were re-analyzed in terms of orbital lag distances, it is possible that the evolution of the lag between the epoch of GUST86 and the present time would be apparent.

### 3. Iapetus Eclipsed by Saturn

Although our observations of 2007 May 5 are the first photometric data set for an eclipse of this satellite by Saturn, Iapetus has previously been observed in the shadow of the planet's rings. Neugebauer et al. (2005) obtained 20 and 2.2  $\mu\text{m}$  measurements revealing that the satellite has a very small thermal inertia.

#### 3.1 Observations

The ingress light curve plotted in Fig. 3a and the schematic of satellite-shadow geometry in Fig. 4 show that the brightness of Iapetus was already declining due to the ring system prior to the planetary eclipse. The eclipse by Saturn itself began with the sharp brightness gradient near UT day fraction 0.087. The equation of the straight line fitted to the data before that time has a slope of -0.6878 and a Y-intercept of 0.1162. Close inspection of the totality phase of the light curve, which began at about 0.102 day, reveals that the observed luminosity averaged -0.0025. Thus, in order to correct the observations for the presence of the ring and the slight negative bias, adjusted luminosities were computed empirically from Eq. 2.

$$L_A = (L_U + 0.0025) / (-0.6878 * T + 0.1162)$$

Eq. 2

where  $L_A$  is the adjusted luminosity,  $L_U$  is the unadjusted luminosity, and  $T$  is the day fraction. The adjusted light curve is shown in Fig. 3b, where the slope and bias have been removed and the observations have been fitted to the model and synthetic light curve described below.

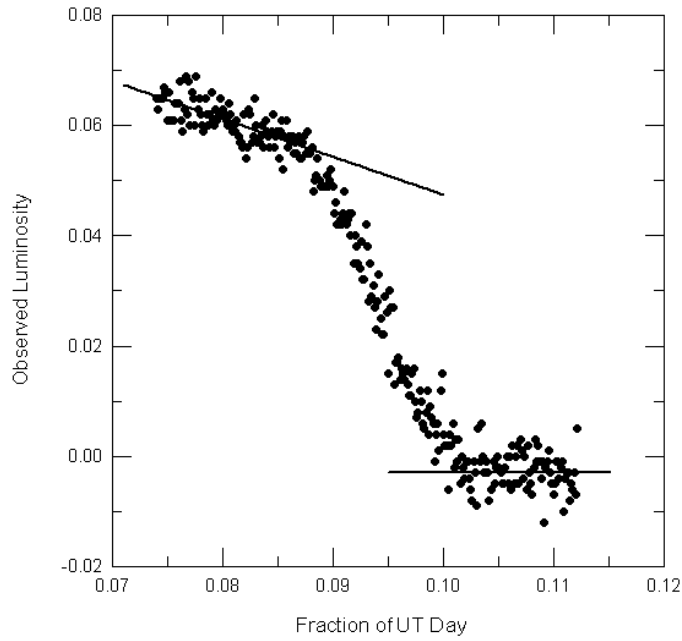


Fig. 3a. The observed light curve of Iapetus where the declining straight line is fitted to luminosities affected by Saturn's ring system before the planetary eclipse began and the horizontal line is fitted to luminosities during total eclipse. The data were acquired by R. Modic, located at 41.564 N and 81.501 W, with a 20 cm telescope through a V filter.

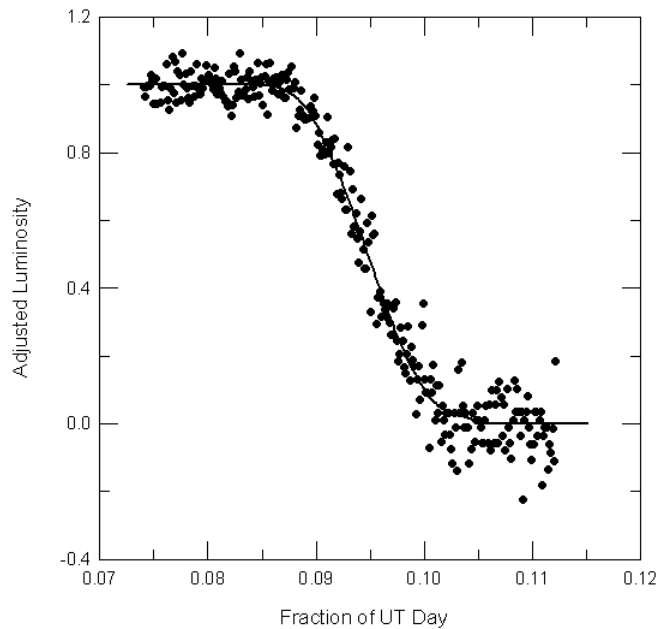


Fig. 3b. The observed light curve was adjusted for the dimming by Saturn's ring and a small negative bias as described in Section 3.1. The solid line is the synthetic light curve described in Section 3.2 and fitted to the brightness-normalized observations as described in Section 3.3.

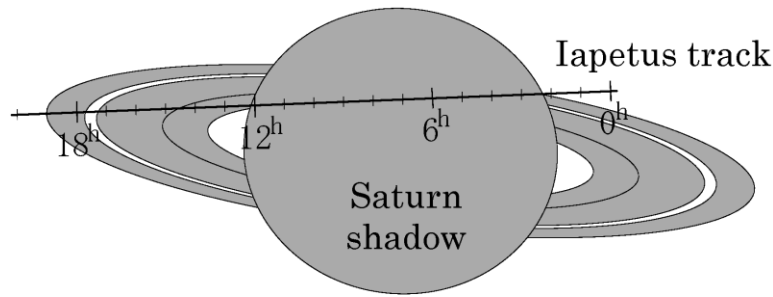


Fig. 4. The line indicating the path of Iapetus through the shadow cast by Saturn reveals that the shadow of

### 3.2 Eclipse model and synthetic light curve

A general eclipse model for saturnian satellites is described by Mallama and Krobusek (1996) and references therein. Additional detailed explanation and illustrations are given by Mallama (2007).

Those methods and results are summarized here and applied to this eclipse of Iapetus.

The dimming of sunlight in a planetary eclipse of a satellite is caused by differential refraction that occurs high above the cloud tops in the milli-bar altitude range of the atmosphere where rays are bent inward toward the shadow center. The intensity of light traversing the atmosphere at lower altitudes becomes fainter because refraction increases progressively with atmospheric density which is exponential. The bending of that ray whose intensity is reduced to exactly half of its original strength is described by Baum and Code (1953). The minimum altitude of this half-intensity ray for Iapetus and Saturn is 240 km above the 1-bar saturnian equatorial radius of 60,268 km (Lindal et al., 1985). The diminished strength of any other refracted ray at the satellite location can be computed by reference to the half-intensity ray. Luminosity in the shadow is a summation over products of limb-darkened solar intensity times the effect of refraction.

In a reference frame where the direction from the center of the Sun to that of the planet defines one axis a straight line from the Sun to the planetary limb is projected outward by 158 km like a truncated cone to the distance of Iapetus in proportion to the angular radius of the planet as seen from the Sun. Additionally, there is one scale height or 55 km of inward bending of the half-intensity ray due to refraction. So, the effective radius for an equatorial eclipse of Iapetus is  $60,268 + 240 + 158 - 55 = 60,611$  km.

In addition to the solar eclipse luminosity described above, the physical size and the BDR characteristics of a satellite must be factored in when computing its integrated brightness in the planetary shadow. The most important aspect of the BDR function for Iapetus is the striking albedo differences between the leading and trailing hemispheres, which was modeled using Eq. 3,

$$A \propto 1 + k * \cos(\gamma)$$

Eq. 3

where  $A$  is albedo,  $k$  is the constant  $-0.98$ , and  $\gamma$  is the angular distance measured on the satellite from the direction of its motion. This cosine characterization produces a sharp brightness boundary as shown in Cassini spacecraft imagery of Iapetus and the numerical value of  $k$  reproduces the rotational light curve observed by Noland et al. (1974). The brightness dichotomy causes an offset of about 200 km between the center-of-brightness of the satellite and its center-of-figure. Limb-darkening and the solar phase effect are also modeled but they produce a much smaller offset.

The synthetic eclipse light curve is generated by evaluating satellite brightness as a function of its position in the solar eclipse space. At any one position it is computed by integrating the product of solar eclipse luminosity and the BDR function over the hemisphere of the satellite turned toward the sensor. Satellite eclipse phase space is a useful construct defined such that phase 0 of an ingress occurs when the leading edge of the satellite is tangent to the geometric penumbra and

phase 1 takes place when its trailing edge is tangent to the boundary between the geometric penumbra and umbra. Because of refraction, the luminosity of the satellite at phase 0 will be less than unity and at phase 1 it will be greater than 0. A half-intensity ray from the center of the solar disk falls on the center of the satellite disk at phase 0.5 in eclipse space, however the integrated luminosity may be greater or less than 0.5 at that location.

In order to correctly represent the geometry of the event the model also requires the solar phase angle of the satellite and the angle at which the shadow crosses the satellite relative to its equator. The synthetic light curve for this event was generated for a solar phase angle of +6.3 degrees (the plus sign indicating illumination of the satellite from the west). The angle at which the shadow of Saturn crossed the globe of Iapetus was measured at 67 degrees to the equator from a diagram computed with the on-line Saturn Viewer developed by Mark Showalter for the PDS Rings Node ([http://pds-rings.seti.org/tools/viewer2\\_sat.html](http://pds-rings.seti.org/tools/viewer2_sat.html)).

### 3.3 Light curve fitting, estimated uncertainty and comparison with ephemerides

Astrometry is derived when the synthetic light curve is fitted to photometric data and establishes the time at which the satellite crossed phase 0.5 in eclipse space. The mathematical procedure is a non-linear problem of the type described by Bevington (1969). In this case the synthetic light curve is parameterized and successive iterations adjust those parameters until the differences between modeled and observed data are reduced to a least squares minimum. The light curve fitting for this eclipse, shown in Fig. 3b, indicates that Iapetus was located at half-phase of ingress in the shadow by Saturn at geocentric UT 02:14:58, while half-luminosity occurred 92 s later.

The uncertainty of the observational result has several components which include the radius of the half-intensity ray, the model that produced the synthetic light curve, scatter in the observations, the radius of Saturn and the photometric correction for the effect of Saturn's ring. Possible errors in the radius of the half-intensity ray have been analyzed by comparing different computational



approaches and the estimated uncertainty is 10 km. The model computes synthetic luminosities very accurately, however the difference between the simple characterization of Iapetus and the true distribution of luminosity across its surface could result in an error estimated to be 20 km. The uncertainty in the timing of half-phase given above due to scatter is 5 s and, at the 3.3 km/s orbital velocity of Iapetus, this equates to 16 km. The error estimate associated with the radius of Saturn is 4 km. The error due to inaccurate correction for the ring eclipse can be estimated from the effect on the half-phase timing when no correction is applied. Taking half of that 65 second difference (since it is very unlikely that the correction is completely in error) and multiplying by orbital velocity gives 107 km. The root-sum-square of all these error components, which represents the overall uncertainty, is 111 km. The corresponding angular uncertainty is 16 mas at the distance from the Sun to Saturn and the timing uncertainty is 34 s.

The most current ephemeris for Iapetus is SPICE (Acton, 1996) kernel SAT288 which was created by R. A. Jacobson. Taking into account all the geometrical and physical factors described in section 3.2, SAT288 indicates an O – C of +16 s. Another orbit model that fit the observed time fairly closely was the TASS 1.6 ephemeris of Vienne and Duriez (1995) for which the O – C was -70 s. The ephemerides of Harper and Taylor (1993) and Dourneau (1993) did not predict the observed time very accurately, as evidenced by the O – C values of +700 and +1200 s, respectively.

#### 4. Conclusions

The photometry from 5 uranian satellite mutual eclipses and occultations observed in 2007 has resulted in astrometry where relative satellite positions are determined to better than 10 milli-arcseconds for most of the events. These observations plus three additional data points from other observers indicate that the along-track component of motion predicted by the SPICE (Acton, 1996) ephemeris URA083, as well as those predicted by Arlot et al. (2006) from the LA06 ephemeris, are significantly more accurate than predictions based on the GUST86 ephemeris made by Christou (2005). However, the actual positions of Ariel, Umbriel, Titania and Oberon are lagging behind their predicted positions for all of the ephemerides. These results may indicate missing perturbation terms in some satellite ephemerides.

The ingress portion of a saturnian eclipse of Iapetus on 2007 May 5 produced an astrometric result with an uncertainty of about 16 milli-arcseconds. The satellite position fits very well with the SPICE ephemeris SAT288 and reasonably well with the TASS 1.6 ephemeris of Vienne and Duriez (1995). Two other ephemerides did not agree as closely with the observed result.

#### Acknowledgment

We are grateful to V. Lainey and an anonymous reviewer of this paper for their excellent comments and recommendations. P. Sada acknowledges Grant No. 52274 from the Consejo Nacional de Ciencia y Tecnología (CONACYT) of Mexico which upgraded equipment at the Universidad de Monterrey Observatory used to obtain some of the data.

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